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Let $n=2$, then $3^2-1^2=2^3$,
" $n=3$, then $6^2-3^2=3^3$,
" $n=4$, then $10^2-6^2=4^3$, or in the series 1, 3, 6, 10, 15, etc., we have the difference of the squares of any two contiguous terms equal a cube.

Second case. Let $\frac{y^3}{x^3}$ be the fraction.. Then $x^3+y^3=n^2$, or $(x+y)(x^2-xy+y^2)=n^2$. If $(x+y)$ be a square, then $x^2-xy+y^2$ will be a square. This is only possible when $x=y$. $\therefore$ the sum of any two equal cubes, the sum of whose roots is a square, will be a square, as $\frac{8^3}{8^3}$, or $\frac{512}{512}$, will be an improper cubic fraction the sum of whose terms will be a square. If $x+y=x^2-xy+y^2$, then $x^3+y^3$ will be a square. This is only possible when $x=2$, $y=1$, and the proper fraction $\frac{1}{8}$ will be cubic and the sum of the terms a square number.

$\therefore (2^3+1^3)=3^2$; also $2^6(2^3+1^3)=24^2$, etc.

Also solved by *P. S. BERG*, *A. L. FOOTE*, *G. B. M. ZERR*, and the *PROPOSER*.

## PROBLEMS.

**34. Proposed by R H. YOUNG, West Sunbury, Pennsylvania.**

Prove (1) that $\frac{n(n+1)(2n+1)}{6}$ is a whole number for all values of $n$; and (2) prove that $\frac{(n-1)n(n+1)}{24}$ is a whole number when $n$ is odd.

**35. Proposed by G. B. M. ZERR, A. M., Ph. D , Vice-President and Professor of Mathematics and Sciences, Inter State College, Texarkana Texas.**

Decompose into the sum of two squares the number $13^2.61^3$.

**36. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find the first six integral values of $n$ in $\frac{n(n+1)}{2}=\square$.

# AVERAGE AND PROBABILITY.

**Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.**

## SOLUTIONS OF PROBLEMS.

**19. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Find the average area of the circle which is the locus of the middle points of all chords passing through a point taken at random in the surface of a given circle.

**II. Solution by JOHN DOLMAN, Jr., Philadelphia, Pennsylvania.**

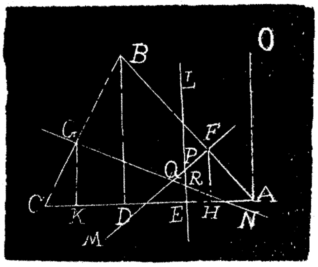
If the random point be taken upon the circumference of a variable circle, concentric with the given circle, the area of the circular "locus" is evidently always one quarter of this variable circle. Again: As the random points are equably distributed over the area of the given circle, the variable circle must always increase in area by equal increments, and, as it varies from zero to the full area of the given circle, its mean area is, of course, half of the given circle. Therefore the mean area of the locus named is one half of one quarter or one eighth of the given circle.

**23. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Find the average area of all the triangles that can be drawn *perpendicular-sided* to a given plane scalene triangle.

**Solution by G. B. M. ZERR, A. M., Ph. D., Vice-President and Professor of Mathematics and Sciences, Inter State College, Texarkana, Texas.**

Let $ABC$, be the given scalene triangle; $EL$, $FM$, $GN$ the perpendiculars to $AC$, $AB$, $BC$ respectively, forming the triangle $PQR$. Let $AC$, $AO$ be the axes of co-ordinates. Draw $BD$, $FH$, $GK$ perpendicular to $AC$.



Let $AD=d$, $BC=e$, $BD=h$, $AC=b$, $AB=c$, $BC=a$, $AE=u$, $AH=v$, $AK=w$.

Then $FH=hv/d$, $GK=\dfrac{h(b-w)}{e}$. $x=u$,

$$y=\frac{hv}{d}+\frac{dv}{h}-\frac{dx}{h},\ y=\frac{h(b-w)}{e}+\frac{(b-d)(x-w)}{h},$$

are the equations to $LE$, $FM$, and $GN$ respectively.

$$\therefore\ \frac{h^2ev+d^2ev+w(h^2d-d^2e+bde)-h^2db}{bde}=\frac{c^2ev+a^2dw-h^2db}{bde}.$$ is the abscissa of $Q$. Let $A_1=$area $PQR$, $\triangle=$ area $ABC$.

$$\therefore\ A_1=\frac{\triangle\,(c^2ev+a^2dw-h^2db-bdeu)^2}{b^2d^2e^2h^2}=\tfrac{1}{2}\,\frac{(c^2ev+a^2dw-h^2db-bdeu)^2}{bd^2e^2h}$$

The limits of $u$ are 0 and $b$; of $v$, 0 and $d$; of $w$, $d$ and $b$.

$$\therefore\ A=\frac{\int_o^b\int_o^d\int_d^b A_1\,du\,dv\,dw}{\int_o^b\int_o^d\int_d^b du\,dv\,dw}=\frac{1}{bde}\int_o^b\int_o^d\int_d^b A_1\,du\,dv\,dw$$

$$=\frac{1}{6a^2b^2d^4e^3h}\int_o^b\int_o^d\{\,(c^2ev+e^2db-bdeu)^3-(c^2ev-h^2bd+a^2d^2-bdeu)^3\,\}du\,dv$$

$$=\frac{1}{24a^2b^2c^2e^4h}\int_o^b\{\,(c^2e+e^2b-beu)^4-(e^2b-beu)^4-(c^2e-h^2b+a^2d-beu)^4$$